More Energy Concepts

8/19/2019

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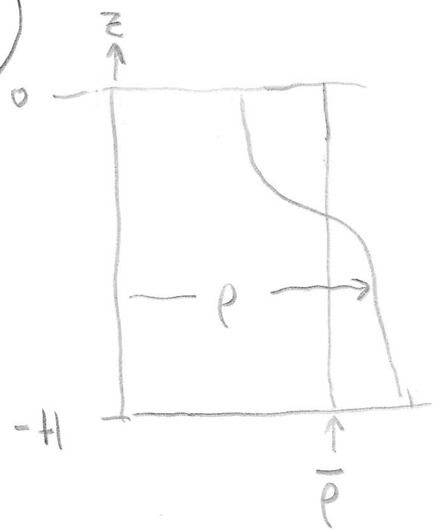


For a stratified water column, how much energy is required to mix to homogeneity?

$PE_V = \rho g z$ = potential energy / unit vol $[J/m^3]$

Change in energy to mix completely:

$$\Delta PE_A = \int_{-H}^{0} \bar{\rho} g z \, dz - \int_{-H}^{0} \rho g z \, dz = \int_{-H} (\bar{\rho} - \rho) g z \, dz$$

PE_A = PE / unit horizontal area

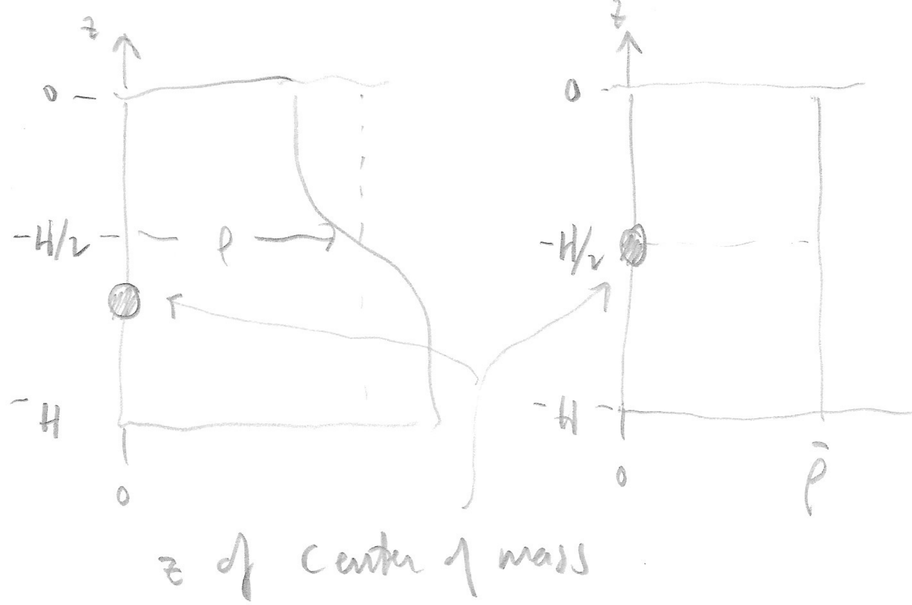
$\frac{\Delta PE_A}{H}$ often called "φ" eg. Simpson (1990)

Useful as a way to quantify stratification

Geyer & Ralston (2011)

ΔPE_A is positive-definite

Corresponds to raising the center of mass of a stratification



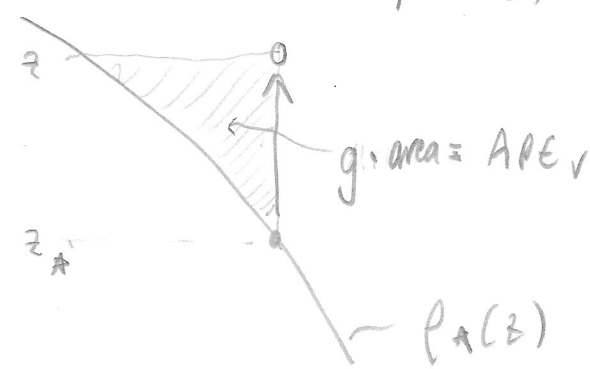
II Available Potential Energy : Baroclinic

APE_v [J/m³]

Work done against buoyancy to push a water parcel from its resting position to its current location

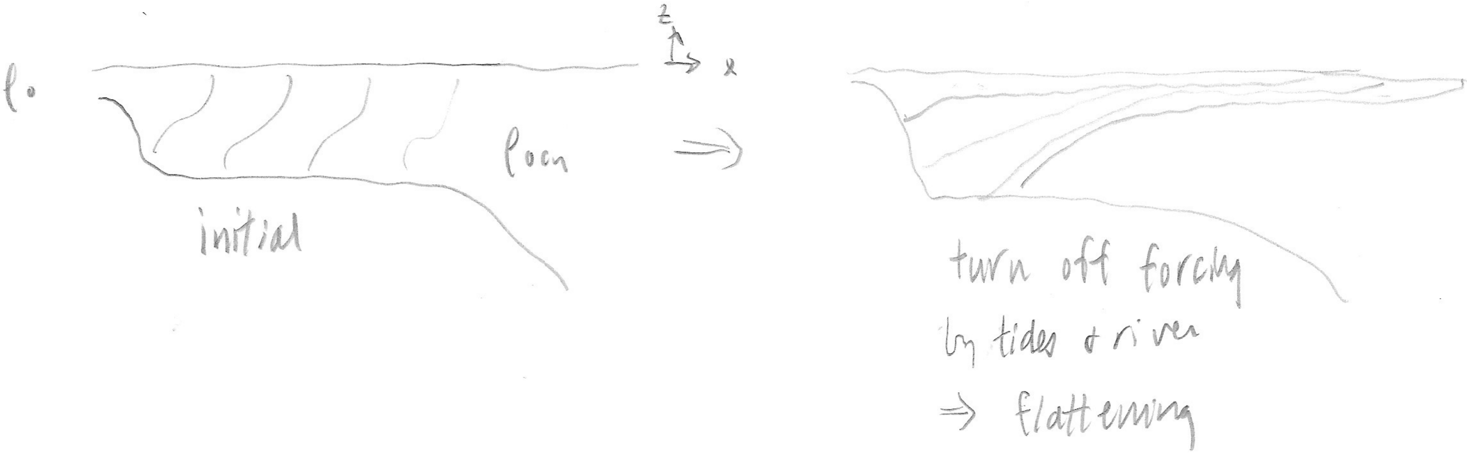
(Holliday & McIntyre 1981)
(MacC & Giddings 2016)

$$(*) \quad APE_v = \int_{z_*}^{z} g(\rho - \rho_*)\, dz$$



- positive definite and local - like $KE_v = \frac{1}{2}\rho_0 \underline{u}^2$
- useful for full systems like exchange flow or upwelling.
- Recall $APE_A^{SW} = \frac{1}{2}\rho_0 g \eta^2$ is the vertically-integrated APE_v of SW flow. The new definition (*) is the same idea applied to deformation of isopycnals instead of the free surface.

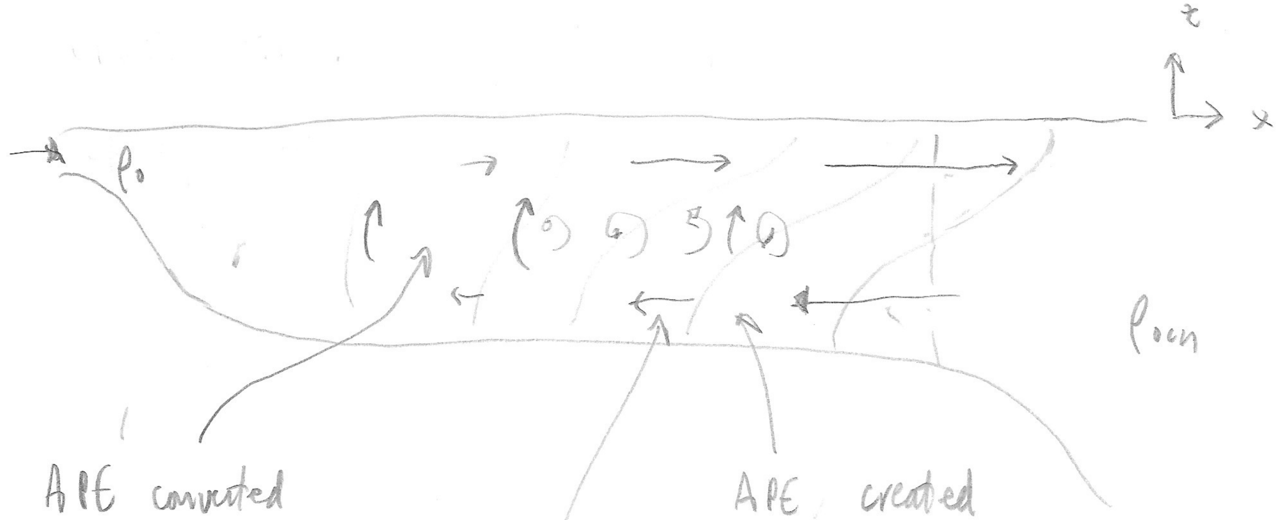
For estuaries we can define a simple resting stratification $\rho_* = \rho_{ocn} = \text{const}$



$$\Rightarrow \quad APE_v = \int_{z_*}^{z} g(\rho - \rho_*)\, dz = \int_0^{z} g(\rho - \rho_{ocn})\, dz = \boxed{g(\rho - \rho_{ocn})z}$$

Positive-definite: $\begin{cases} (\rho - \rho_{ocn}) \text{ negative} \\ z \text{ negative} \end{cases}$ ✓

Estuary tidally averaged energy system



APE converted
to KE by
vertical velocity

APE created
by mixing

Converted APE appears
as the KE of the
exchange flow

$$\Delta PE_A = -g \left\{ \int_{-H}^{-H/2} (\cancel{\bar{\rho}} + \Delta\rho/2)\, z\, dz + \int_{-H/2}^{0} (\cancel{\bar{\rho}} - \Delta\rho/2)\, z\, dz - \cancel{\int_{-H}^{0} \bar{\rho}\, z\, dz} \right\}$$

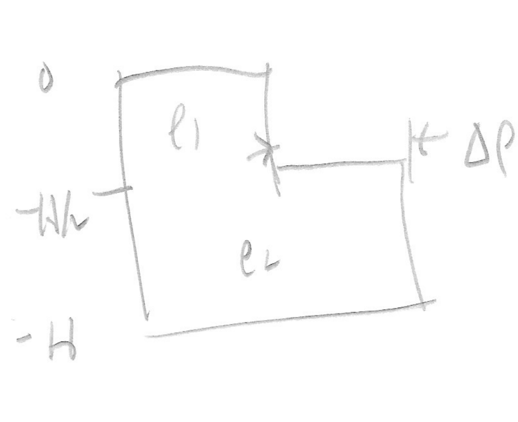
$$= -g \frac{\Delta\rho}{2} \left[\frac{1}{2} z^2 \Big|_{-H}^{-H/2} - \frac{1}{2} z^2 \Big|_{-H/2}^{0} \right]$$

$$= -\frac{g\Delta\rho}{4} \left[\underbrace{\frac{H^2}{4} - H^2 + \frac{H^2}{4}}_{-\frac{H^2}{2}} \right] = \frac{\rho_0}{8} H^2 \frac{g\Delta\rho}{\rho_0} = \boxed{\frac{\rho_0 H}{2} c^2 = \Delta PE_A}$$

$$c^2 = \frac{g\Delta\rho}{\rho_0} \frac{H^2/4}{H} = \frac{g\Delta\rho}{\rho_0} \frac{H}{4}$$

Answer to question

$\Delta PE_A \uparrow$



Forming equations for APE_v + KE_v (estuarine limit: $\rho_* = \rho_{ocn}$, $z_* = 0$)

$APE_v = g(\rho - \rho_{ocn})z$

$KE_v = \frac{1}{2}\rho_0 \underline{u}^2$

$\boxed{\underline{x}\text{ mom}}$ $\quad \underline{u}_t + \nabla \frac{1}{2}\underline{u}^2 + (\underset{\sim}{\omega} + f\hat{k}) \times \underline{u} = -\frac{1}{\rho_0}\nabla p - \hat{k}\frac{g\rho}{\rho_0} + \overset{\circ}{\underline{u}}$

$\sim \frac{\partial}{\partial z}\left(A\frac{\partial u}{\partial z}\right)$ etc.

$\boxed{\text{mass}}$ $\quad \nabla \cdot \underline{u} = 0$

$\boxed{\rho}$ $\quad \frac{D\rho}{Dt} = \overset{\circ}{\rho} \leftarrow \approx \frac{\partial}{\partial z}\left(K\frac{\partial \rho}{\partial z}\right)$

KE_v: $\rho_0 \underline{u} \cdot \boxed{\underline{x}\text{ mom}} \Rightarrow \frac{D}{Dt} KE_v = -\nabla \cdot \underline{u}p - \rho g w + \rho_0 \underline{u} \cdot \overset{\circ}{\underline{u}}$

adding $0 = -\rho_{ocn} g w + \rho_{ocn} g w$

$$\Rightarrow \boxed{\frac{D\,KE_v}{Dt} = -\nabla\cdot(\underline{u}P) - (\rho - \rho_{ocn})gw + \rho_0 \underline{u}\cdot\overset{\circ}{\underline{u}}}$$

Rate of change of KE v following a parcel = Convergence of Pressure Work + Conversion from APE v + Dissipation

where $\nabla P = \nabla p + \hat{k} g \rho_{ocn}$

$-\nabla P = -\hat{i} p_x - \hat{j} p_y - \hat{k}\, g(\rho_{ocn} - \rho) + \hat{k}\, g(\rho - \rho_{ocn})$

APE_v: $g z \boxed{\rho} \Rightarrow \dfrac{D}{Dt}(\rho g z) - \rho g w = g z \overset{\circ}{\rho}$

add $g z \left(\dfrac{D \rho_{ocn}}{Dt} = 0 \right)$ Note $\dfrac{D}{Dt}(\rho g z) = g z \dfrac{D\rho}{Dt} + \rho g \overbrace{\dfrac{Dz}{Dt}}^{w}$

$$\Rightarrow \underbrace{\frac{D}{Dt} APE_v}_{\text{Rate of change of } APE_v \text{ following parcel}} = \underbrace{(\rho - \rho_{ocn}) g w}_{\text{Conversion to } KE_v} + \underbrace{g z \overset{\circ}{\rho}}_{\text{Mixing.}}$$

Note on <u>mixing</u> $g z \overset{\circ}{\rho} \equiv g z \dfrac{\partial}{\partial z}\left(K \dfrac{\partial \rho}{\partial z}\right)$

$$= \underbrace{\frac{\partial}{\partial z}\left(g K z \frac{\partial \rho}{\partial z}\right)}_{\text{vertical } \int dz = 0} \underbrace{- g K \frac{\partial \rho}{\partial z}}_{\text{"buoyancy flux"}}$$

Note on <u>dissipation</u> $\rho_0 \underline{u} \cdot \overset{\circ}{\underline{u}} \cong \rho_0 \left[u \dfrac{\partial}{\partial z}\left(A \dfrac{\partial u}{\partial z}\right) + v \text{ term} \right]$

$$= \frac{\partial}{\partial z}\left(A \tfrac{1}{2} \rho_0 u^2\right) - \rho_0 A \left(\frac{\partial u}{\partial z}\right)^2$$